

# IMO 2007 Vietnam

## Day 2

**Problem 4** In triangle  $ABC$  the angle bisector at vertex  $C$  intersects the circumcircle again at  $R$  and the perpendicular bisectors of sides  $BC$  and  $AC$  at  $P$  and  $Q$  respectively. The midpoint of  $BC$  is  $S$  and the midpoint of  $CA$  is  $T$ . Prove that the triangles  $RQT$  and  $RPS$  have the same area.

**Problem 5** Let  $a$  and  $b$  be positive integers. Show that if  $4ab - 1$  divides  $(4a^2 - 1)^2$ , then  $a = b$ .

**Problem 6** Let  $n > 1$  be an integer. Consider

$$S = \{(x, y, z) : x, y, z \in \{0, 1, \dots, n\}, x + y + z > 0\}$$

as a subset of three-dimensional Euclidean space. Determine the smallest number of planes the union of which contains  $S$  but does not include  $(0, 0, 0)$ .

*Each problem is worth 7 points*

*Time allowed:  $4\frac{1}{2}$  hours*