

13 July 2006

Problem 4. Determine all pairs (x, y) of integers satisfying the equation

$$1 + 2^x + 2^{2x+1} = y^2.$$

Problem 5. Let P be a polynomial of degree $n > 1$ with integer coefficients and let k be any positive integer. Consider the polynomial $Q(x) = P(P(\dots P(P(x)) \dots))$, with k pairs of parentheses. Prove that Q has no more than n integer fixed points, i.e. integers satisfying the equation $Q(x) = x$.

Problem 6. To each side a of a convex polygon we assign the maximum area of a triangle contained in the polygon and having a as one of its sides. Show that the sum of the areas assigned to all sides of the polygon is not less than twice the area of the polygon.

Time allowed: 4 hours 30 minutes