IMO 2005 Mexico

Day 2

Problem 4 Determine all positive integers relatively prime to all the terms of the infinite sequence

$$a_n = 2^n + 3^n + 6^n - 1.$$

Problem 5 Let $ABCD$ be a fixed convex quadrilateral with $BC = DA$ and BC not parallel with DA. Let two variable points E and F lie on the sides BC and DA respectively, and satisfy $BE = DF$. The lines AC and BD meet at P, the lines BD and EF meet at Q, the lines EF and AC meet at R.

Prove that as E and F vary, the circumcircles of PQR have a common point other than P.

Problem 6 In a mathematical competition in which 6 problems are posed to all participants, every two of these problems were solved by more than $2/5$ of the contestants. Moreover, no contestant solved all 6 problems. Show that there are at least 2 contestants who solved exactly 5 problems.

Each problem is worth 7 points

Time allowed: $4\frac{1}{2}$ *hours*